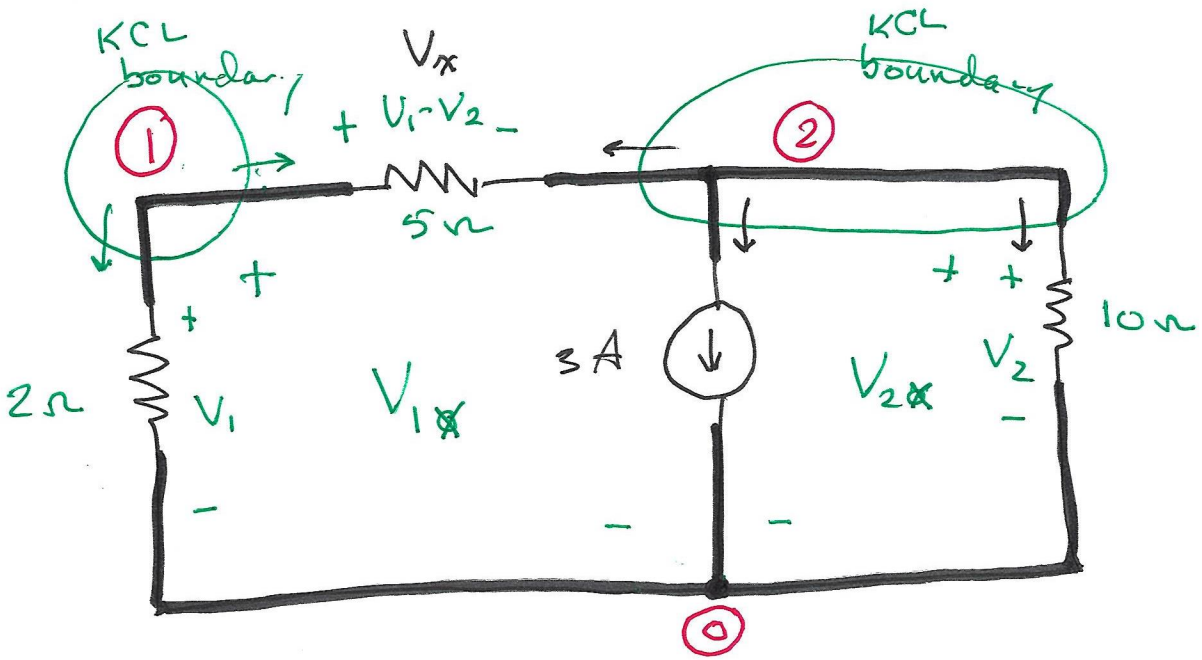
Nodal Analysis

Based on KCL.

Find all node voltages.



3 nodes

1. Choose a reference node and label it ⓪.

At node 1: $+\frac{V_1 - V_2}{5} + \frac{V_1}{2} = 0$

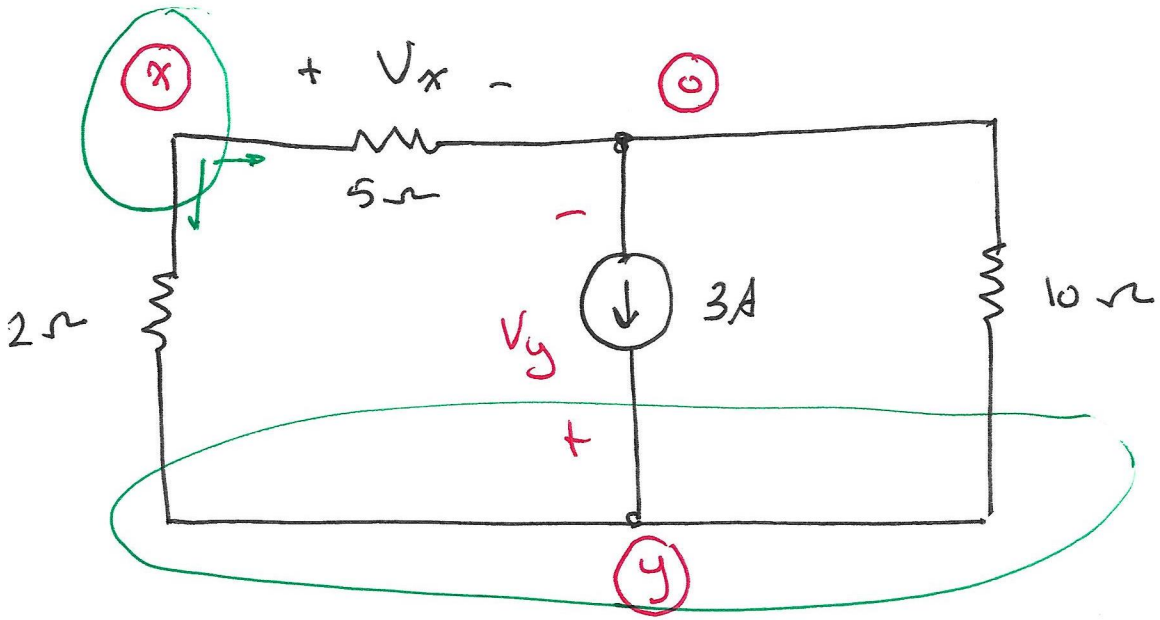
At node 2: $\frac{V_2 - V_1}{5} + 3 + \frac{V_2}{10} = 0$

In matrix form:

$$\begin{bmatrix} \frac{1}{5}+\frac{1}{2} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{1}{5}+\frac{1}{10} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \end{bmatrix}$$

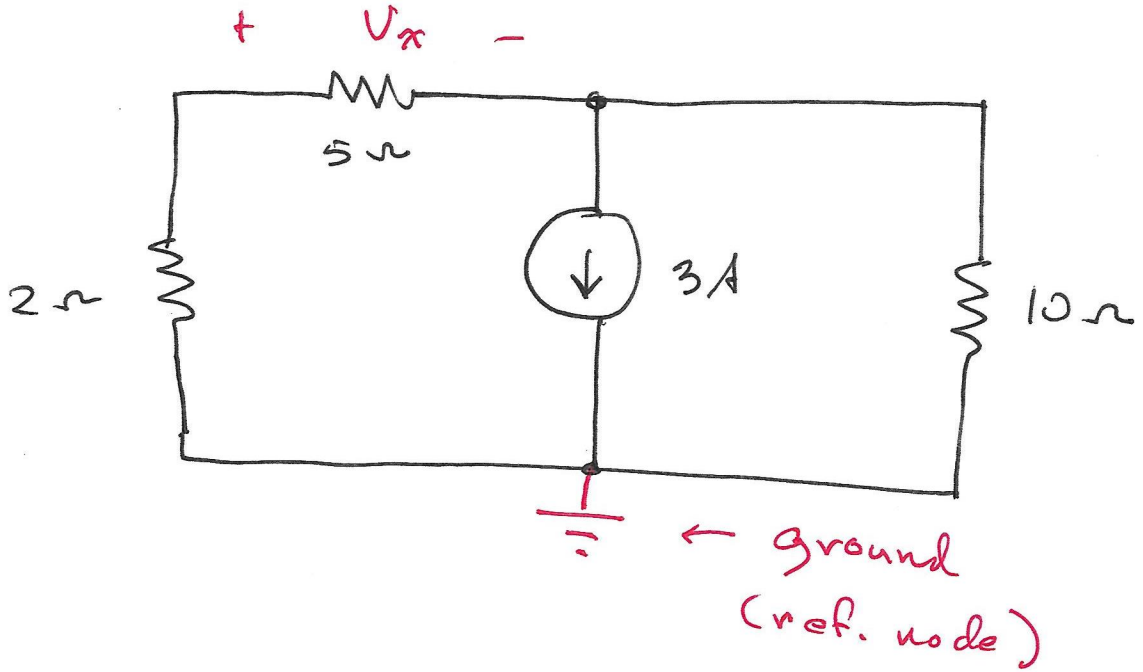
Solving yields V_1 and V_2.

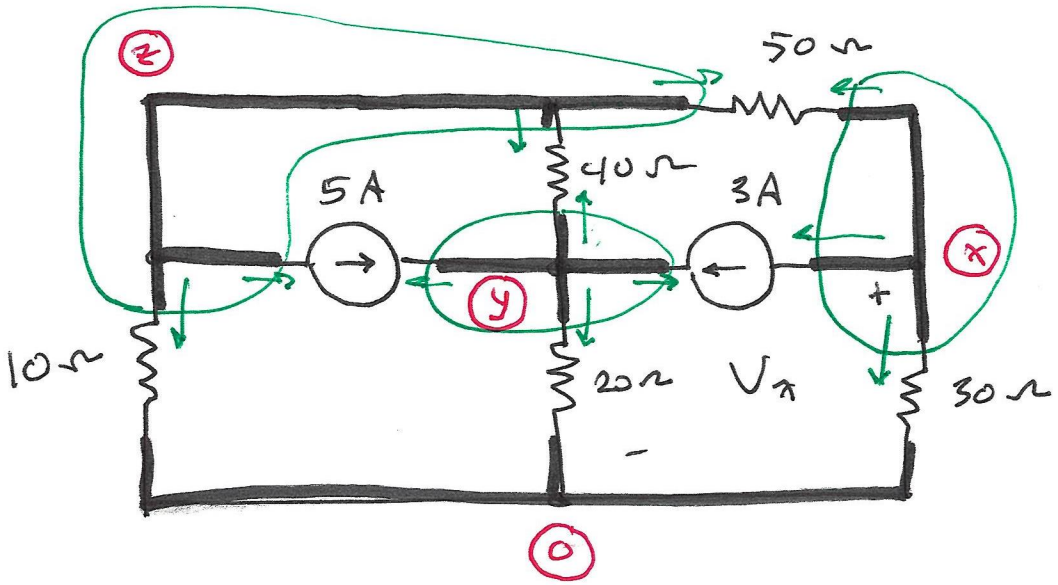
Then $V_x = V_1 - V_2$



$$\frac{V_x}{5} + \frac{V_x - V_y}{2} = 0 \qquad \text{at node } (x)$$

$$\frac{V_y - V_x}{2} - 3 + \frac{V_y}{10} = 0 \qquad \text{at node } (y)$$


+ V_x −
5 Ω
2 Ω
3 A
10 Ω
← ground
(ref. node)



Write node eqns. to solve for V_x.

3 nodes other than the ref. node

$\Rightarrow$ 3 node equations.

$$\frac{V_x}{30} + 3 + \frac{V_x - V_z}{50} = 0 \qquad \text{(KCL at node x)}$$

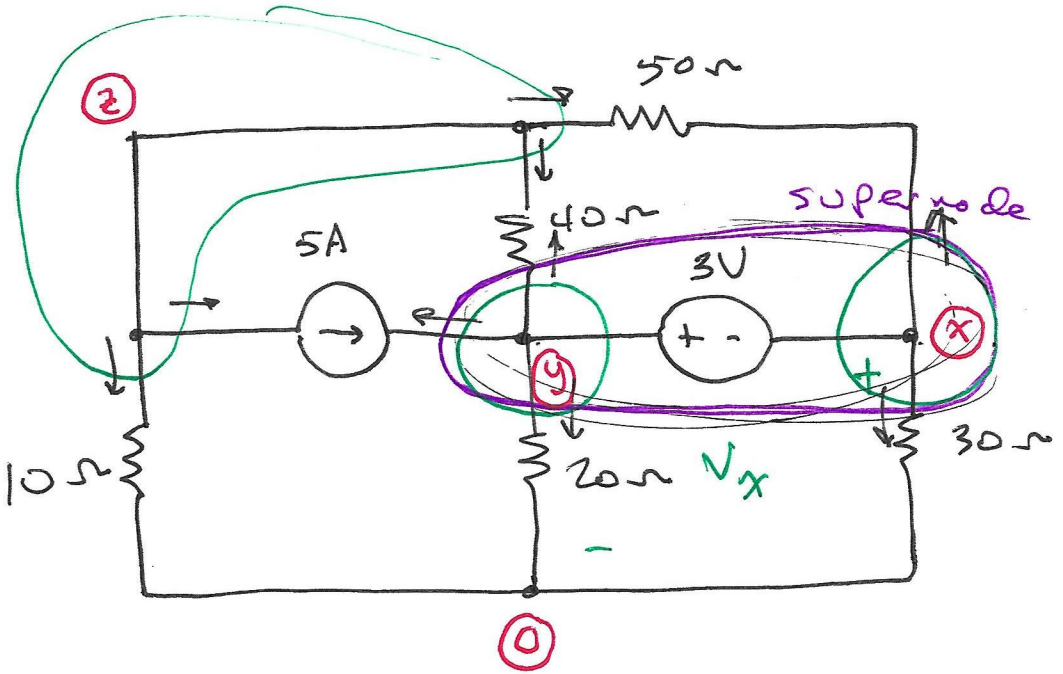
$$\frac{V_y - V_z}{40} - 5 - 3 + \frac{V_y}{20} = 0 \qquad \text{(KCL at node y)}$$

$$\frac{V_z}{10} + 5 + \frac{V_z - V_y}{40} + \frac{V_z - V_x}{50} = 0 \qquad \text{(KCL at node z)}$$

In matrix form:

$$\begin{bmatrix} \frac{1}{30}+\frac{1}{50} & 0 & -\frac{1}{50} \\ 0 & \frac{1}{40}+\frac{1}{20} & -\frac{1}{40} \\ -\frac{1}{50} & -\frac{1}{40} & \frac{1}{10}+\frac{1}{40}+\frac{1}{50} \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \begin{bmatrix} -3 \\ 8 \\ -5 \end{bmatrix}$$

What if the circuit contains a voltage source?

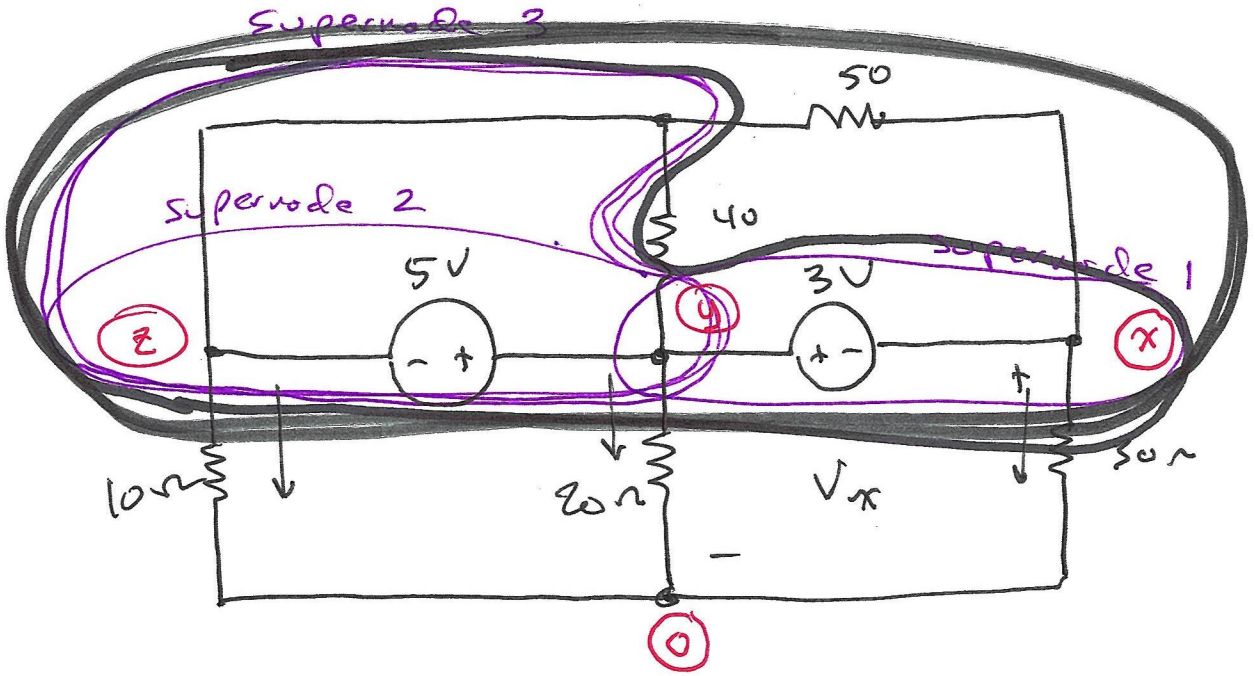


Solve for V_x using the nodal method.

$$V_y - V_x = 3 \qquad \text{(constraint equation)}$$

$$\frac{V_y - V_z}{40} - 5 + \frac{V_y}{20} + \frac{V_x}{30} + \frac{V_x - V_z}{50} = 0 \qquad \text{(KCL for supernode)}$$

$$\frac{V_z - V_x}{50} + \frac{V_z - V_y}{40} + 5 + \frac{V_z}{10} = 0 \qquad \text{(KCL for node z)}$$



$V_y - V_x = 3$ (constraint eqn. for supernode 1)

$V_y - V_z = 5$ (constraint eqn. for supernode 2)

$\frac{V_z}{10} + \frac{V_y}{20} + \frac{V_x}{30} = 0$ (KCL for merged supernode 3)

$$
\begin{array}{l}
 \\
\text{eqn 1} \\
\text{eqn 2} \\
\text{eqn 3}
\end{array}
\begin{array}{c}
\begin{array}{ccc} v_x & v_y & v_z \end{array} \\
\begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \\ \frac{1}{30} & \frac{1}{20} & \frac{1}{10} \end{bmatrix}
\end{array}
\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 0 \end{bmatrix}
$$

Exam # 1 available Sept. 23

due by 9:15 AM Sept. 28

open book, open notes, computer, calculator,

No class meeting on Sept. 28